Scientific and Engineering Notations

Scientific Notation

Numbers that occur in the science and engineering and fields vary from the very large to the very small. In order to make the mathematical operations to be as simple as possible, mathematicians have developed a notation that utilizes the properties of power of ten.

The properties are represented in the following manner:

\[ 1 = 10^0 \]

\[ 10 = 1 \times 10^1 \]

\[ 0.1 = \frac{1}{10} = 1 \times 10^{-1} \]

\[ 100 = 1 \times 10^2 \]

\[ 0.01 = \frac{1}{100} = 1 \times 10^{-2} \]

\[ 1,000 = 1 \times 10^3 \]

\[ 0.001 = \frac{1}{1,000} = 1 \times 10^{-3} \]

\[ 10,000 = 1 \times 10^4 \]

\[ 0.0001 = \frac{1}{10,000} = 1 \times 10^{-4} \]

\[ 100,000 = 1 \times 10^5 \]

\[ 0.00001 = \frac{1}{100,000} = 1 \times 10^{-5} \]

Note that numbers greater than one have positive exponents while numbers less than one have negative exponents and the number equal to one has an exponent equal to zero. Another advantage of scientific notation is that it clearly expresses the amount of significant figures (reliably known digits within a number).

Below are examples of expressing numbers less than one.

**EXAMPLE 1**

Express 0.032 in scientific notation.

*Solution:*

\[ 0.032 = \frac{32}{1,000} = 3.2 \times 10^{-2} \]

The number 3.2 is referred to as the characteristic, the number to the right of the decimal point is called the mantissa and the number \(-3\) is called the exponent. If the characteristic has only one character to left of the decimal point, the number is said to be expressed in scientific notation.

**EXAMPLE 2**

Express 0.046 in scientific notation.

*Solution:*

\[ 0.046 = \frac{4.6}{100} = 4.6 \times 10^{-2} \]
EXAMPLE 3
Express 0.000761 in scientific notation.

Solution:

\[0.000761 = \frac{7.61}{10,000} = 7.61 \times 10^{-4}\]

EXAMPLE 4
Express -0.000761 in proper scientific notation.

Solution:

\[-0.000761 = \frac{-7.61}{10,000} = -7.61 \times 10^{-4}\]

Below are examples of expressing numbers larger than one.

EXAMPLE 5
Express 3570 in scientific notation.

Solution:

\[3570 = 3.57 \times 1,000 = 3.57 \times 10^3\]

EXAMPLE 6
Express 45,700 in scientific notation.

Solution:

\[45,700 = 4.57 \times 10,000 = 4.57 \times 10^4\]

Engineering Notation

In order to simplify communications when referring to large and small numbers, engineers have developed a written form of abbreviations as described in the table below. Note that exponents occur only in multiples of positive and negative 3. Engineering notation only allows multiples of 3 as exponents.

<table>
<thead>
<tr>
<th>Power of 10</th>
<th>Prefix</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>Tera</td>
<td>T</td>
</tr>
<tr>
<td>$10^9$</td>
<td>Giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^6$</td>
<td>Mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$</td>
<td>Kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>Milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>Micro</td>
<td>µ</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>Nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>Pico</td>
<td>p</td>
</tr>
</tbody>
</table>
**Example 7**
Express 0.035 seconds in terms of milliseconds.

*Solution:*

\[
0.035 \text{ seconds} = \frac{35}{1,000} \text{ seconds} = 35 \times 10^{-3} \text{ seconds} = 35 \text{ milliseconds} = 35 \text{ ms}
\]

**Example 8**
Express 12,300 meters in terms of kilometers.

*Solution:*

\[
12,300 \text{ meters} = 12.3 \times 1,000 \text{ meters} = 12.3 \times 10^3 \text{ meters} = 12.3 \text{ kilometers} = 12.3 \text{ km}
\]