

Optimal Tourney

Dragon attacks may now sound like fairytale material, but once they were a clear and present danger, which required personal attention of King Arthur. When the king received reliable reports of dragon sightings, he would send one of the Knights of the Round Table to find the beast and drive it out of the king's domain.

Since facing a dragon was a very knightly thing to do, the king usually got a lot of volunteers and had to select the most skilled fighter among them for the mission. The selection was based on a tourney, that is, a series of non-mortal fights. Each fight involved two knights and revealed the more skilled among the two. For n volunteers, the tourney consisted of $n - 1$ fights. Specifically, the knights formed a waiting line, and then the first two battled each other, the winner of the first fight battled the third knight in the line, the winner of the second fight battled the fourth knight, and so on. The winner of the last fight was the most skilled knight, who got the honor of facing the dragon.

One day, King Arthur received simultaneous reports of two dragons in different parts of the country, and he thus had to select the two most skilled knights. A straightforward selection procedure would involve $2 \cdot n - 3$ fights; specifically, the king could run the usual tourney among n volunteers to select the most skilled, and then run another tourney among the remaining $n - 1$ volunteers to select the second best. He talked it over with Merlin, who had extensive knowledge of math and algorithm theory, and Merlin pointed out that the king could select the two best knights in fewer fights.

Your task is to figure out the minimal number of one-on-one fights required for selecting the best and the second best among n volunteers. You may assume that a fight always reliably reveals the more skilled among the two participants, and that all knights have different skill levels, which means that a fight never ends in a draw.

1. Determine the minimal required number of fights for the following cases, and fill in the empty cells of the table.

Number of knights (n)	Minimal required number of fights
3	3
5	2
7	
9	
23	
239	

2. Give a formula for the minimal required number of fights, in terms of the number n of knights, and briefly explain your answer.