Designing Educative Guides: Reconceptualizing Teacher's Role in Teacherless Cognitive Tutor-based Robotics Instruction[1]

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1. INTRODUCTION

One research objective of the Robot Algebra Project (RAP) is to develop and/or enhance the algebraic (proportional reasoning) abilities of middle school students. This is done by simulating one-on-one human tutoring through the use of personalized, robotics-focused curriculum within a Cognitive Tutor, an Intelligent Tutoring System-based Learning Environment (ITSLE) – see Roll et al., 2007; Ritter et al, 2007; Moos and Azevedo, 2009; & Slavin, Lake and Groff, 2009. This approach enables each student to proceed through the curriculum at his/her own learning pace. Proportional Reasoning is foregrounded in the robotics curriculum because this provides an interesting, relevant and challenging context for students to engage with the embedded mathematics.

The second RAP research objective is to develop educative teacher guides that would not only facilitate teacher learning, but also catalyze desired student learning outcomes, with respect to procedural and conceptual understanding in reasoning proportionally. Therefore the RAP goal is to develop and assess the efficacy of student and teacher materials that would enhance both teacher and student learning in proportional reasoning.

Using the RAP as the example, the purpose of this paper is to define the role of the instructor in a computer-based learning environment. We view this as a necessary and critical first step toward the design of educative teacher guides for computer-based learning environments for several reasons. First, the student materials that form the core of conventional teacher educative guides are most often inquiry-based (i.e., the goal is student-constructed conceptual knowledge – Remillard, 2005; see also Davis & Krajcik, 2005; Ball & Cohen, 1996); in most ITSLEs, on the other hand, the goal is the development of procedural fluency, not conceptual understanding. Second, most conventional educative teacher guides are meant to be used in classroom settings in which a certified teacher leads a group of students through a topic or an investigation using the inquiry-based student materials.

These conditions are not present in ITSLEs where instruction is designed as an individually paced learning experience for students, and where the vast majority of learning is theorized to take place between the student and the ITSLE. In such environments, the role of the teacher is ill-defined, at best. Finally, in the RAP, the designers are committed to assisting not only certified teachers but also instructors in informal settings (e.g., Boys & Girls Clubs) with implementation. Such individuals most likely have more variable levels of content knowledge than certified teachers and educative teacher guides will need to take this into account.

Towards this end, we present a model in which we conceptualize the role of the teacher *or informal instructor* in a learning environment that is largely intended to be a *teacherless* ITSLE model, and in which the most likely learning outcome is *procedural fluency.* The goal is to conceptualize teacher's role, such that it would be predictive of teacher-facilitated learning outcomes. We aim to answer the question: How would teacher's role need to be conceptualized in such a learning environment or instructional system, to realize a student learning outcome that is not only procedural, but also encompasses conceptual understanding? Further, how may this conceptualization be extended to include instructors, with variable teaching knowledge and capacity, who teach in informal learning environments?

The *contributions* of this study include:

- Conceptualizing the role of the teacher in a basically *teacherless* computer-based instructional system (i.e., ITSLE);
- Framing how the role of the teacher may not only help in catalyzing the procedural knowledge intent of the base (student) materials but also stimulate conceptual understanding-based outcomes;
- Develop a heuristic for designing educative teacher guides for ITSLEs with the goal of catalyzing procedural and conceptual understanding, and in a way that expands the role of the teacher, such that it includes instructors in both formal and informal learning sites.

2. BACKGROUND

In this section, we briefly describe the robotics curriculum, our rationale for focusing on proportional reasoning, and enabling related instruction through the Cognitive Tutor ITSLE.

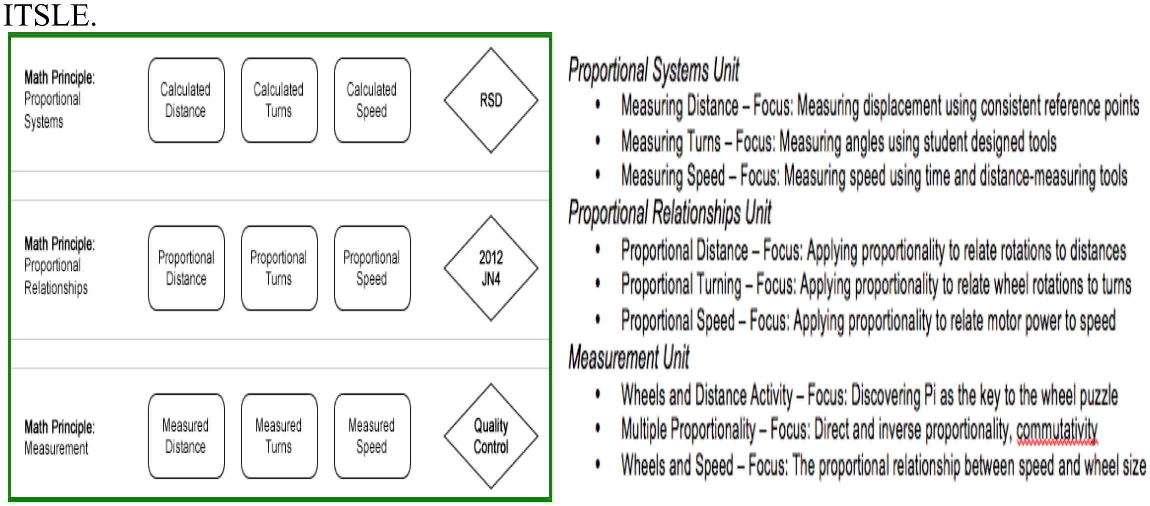

Figure 1. Description of the robotics curriculum.

2.1 The Robotics Curriculum

In the RAP, a core mathematical concept, proportional reasoning, is foregrounded in a robotics curriculum that is in turn delivered through a personalized computer-based Tutor. The robotics curriculum (the student materials) is composed of three modules, Measurement, Proportional Relationships, and Proportional Systems, with each module consisting of three sub-modules (Figure 1). The curriculum builds on the fundamental robot movement concepts: straight, turning, speed and path planning, using proportional reasoning-based strategies.

During an implementation at a formal or informal learning site, a student logs in with a unique ID, and is then taken to the Tutor robotics programming interface. For example, in the Proportional Distance module, the student is presented with a series of tasks where s/he has to determine the proportional distance a robot needs to move, using either a unit rate or scalar (functional) strategy (Figure 3). The calculated value is then input into the appropriate slot in the programming interface to get a physical robot that each student is assigned to perform the specified movement or tasks (Figure 2). The Tutor-robotics interface is designed such that it presents minimal challenges for novice student students or teachers, i.e., programming ability is not a barrier (Silk et al., 2010).

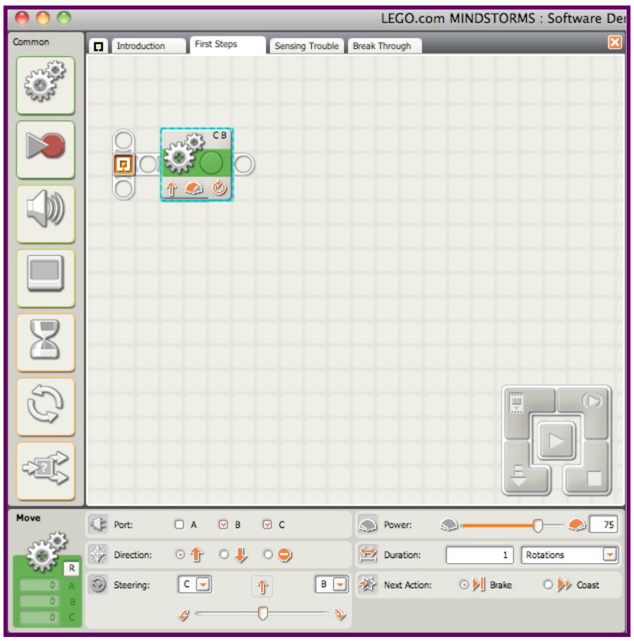

Figure 2. The robotics programming interface.

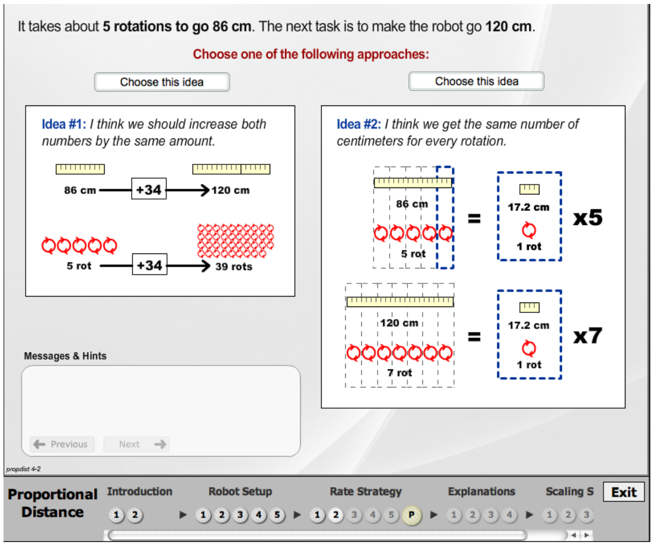

Figure 3. Illustration of the introduction to the unit rate page in Tutor.

2.2 Why Proportional Reasoning?

In the RAP, the core math concept that is foregrounded in the robotics curriculum is proportional reasoning. This is because the ability to reason proportionally is critical to the development of mathematical and scientific thinking (NCTM, 2000; Lesh, Post & Behr, 1988). However, research shows that both teachers and students do not know how to reason proportionally, and also many adults do not adopt proportional reasoning skills (Lamon, 2007). Further, research shows that interventions aimed at middle school students are most effective for correction and laying the foundation for the developing students capacity to use proportional reasoning strategies (Ben-Chaim et al., 1998).

Mathematically, a proportional relationship is defined by the equation $y= Mx$**,** where m (slope or gradient) defines the multiplicative relationship between the quantities y and x. Research has shown that many teachers and students typically solve problems involving proportional situations using the cross-multiplication strategy due to a limited understanding (Lesh, Post & Behr, 1988). Some also to resort to using guess and check strategies, with no intention of developing the requisite understandings (Silk & Schunn, 2011). These approaches represent the lowest form of proportional reasoning ability. Teachers and/or students with more advanced proportional reasoning abilities will be able to use multiple strategies, including unit rate and scalar factor, to solve a range of proportional reasoning problems – missing value, comparison, etc.; and be able to distinguish proportional (i.e., multiplicative) from non-proportional (i.e., additive) situations (Lamon, 2007).

2.3 The Teacherless Tutor

Intelligent Tutoring Systems-based Learning Environments (ITSLEs), as used here, refer to a subset of computer-assisted instruction (CAI) technology programs that aim to enhance student learning of related content by "*substantially replacing the teacher with self-paced instruction on the computer (e.g., Cognitive Tutor, I Can Learn)"* (Slavin, Lake & Groff, 2009, p. 842). For example, the designers of the Cognitive Tutor (Tutor) described it as a "relatively teacherless and isolated model (Anderson et al., 1995, p. 199). Hence the Tutor is an example of a teacherless CAI, as its design rationale is the simulation of one-to-one human tutoring that research shows has an unparalleled impact on student learning (Bloom, 1984).

The ITSLEs largely replace the 'human teacher' by "providing core instruction, opportunities for practice, assessment, and prescription, all tailored to the needs of each student. The teacher's role is [ideally] to circulate among students, provide encouragement, and answer questions but not to provide extensive direct instruction" (Slavin, Lake & Groff, 2009, p. 858). The Tutor ITSLE simulates the 'human teacher' role through the monitoring of student *performance* and *learning* (Koedinger & Corbett, 2006, p. 62). This monitoring includes the provision of real time feedback, corrective help or hint utility, and scaffolded task mastery learning progression facilities (Ritter at al, 2007, p. 252).

Further, the Tutor offers three benefits: **(1)** It is based on a validated learning theory model (Anderson et al., 1995), and it has been used in over 2000 schools (Roll et al, 2007), with proven enhanced learning outcomes (Slavin, Lake & Groff, 2009; Koedinger & Corbett, 2006); **(2)** the Tutor interface allows the simulation of the robotics (programming and robot motion) context; **(3)** The Tutor focuses on the use of two proportional reasoning strategies – unit rate and scalar factor. This focus is to develop students' procedural fluency in the use of proportional reasoning strategies, which a reliance on guess and check strategies inhibits (Silk & Schunn, 2011).

The unit rate strategy is a between-ratio approach, while scalar (functional) strategy is a within-ratio approach. As shown in Figure 3, students are prompted to learn how to figure and use a unit rate to solve problems such as: if a robot goes 86 cm with 5 wheel rotations, how many wheel rotations will it take to move the robot 120 cm? The student is prompted to figure out that the relationship between wheel rotations and distance (between-ratio approach) is 17.2. He or she then uses this unit rate to figure how many wheel rotations it will take to go 120 cm by dividing 120 by 17.2. Figure 3 illustrates how students are prompted to use a scaling strategy. Students are shown that going from a movement of 10 cm to a movement of 20 cm entails a doubling (within-ratio approach) or a scale factor of 2. Thus, if it took 3 rotations to move 10 cm, it will take 6 rotations to move 20cm.

The sequencing of teaching and practice problems used in the Tutor approximates an environment for learning procedural fluency. Once a student has learned how to solve problems using each of the above approaches, he or she is given more problems of the same type, problems that can be solved using the same procedural steps, thereby giving student practice in developing fluency but not challenging them to think conceptually.

In the next section, we present the model and describe its two main constructs.

3. THE MAIN MODEL CONSTRUCTS

We present a model where teachers' conceptions of their roles during RAP implementations are predictive of the learning outcomes with respect to the Mathematical Strands of Proficiency (NRC, 2001). The model therefore consists of two major constructs: (1) teacher's role and (2) the related learning outcomes. There is a third set of constructs, i.e., control variables (e.g., teacher knowledge, student and learning site attributes), but this only influences the degree and quality (i.e., model fidelity) of the teacher-facilitated learning outcomes realized during RAP implementations. Hence they will be presented only as supplementary or influencing factors. However, they would be important considerations in the design and development of educative guides.

The model is based on the *research question*: How may teacher's role be conceptualized in a teacherless ITSLE that by design facilitates the acquisition of procedural fluency, to realize a student learning outcome that includes both procedural and conceptual understanding, in both formal and informal learning environments? Before we describe the two main constructs, there is a need to explicate the assumptions underpinning the model.

First, the model assumes that the Tutor content – the base (student) materials can, as a minimum, facilitate the achievement of procedural fluency, i.e., the baseline for the model is that the student learning materials should be able to facilitate procedural fluency or knowledge acquisition (Davis & Krajcik, 2005). Second, since the Tutor content is procedural fluency-focused, a focus on the Tutor as the predominant 'teacher', would by extension largely preclude the realization of understanding-focused outcomes. Third, the teacher's role is essential to helping students to make connections between the relevant concepts, both through and external to the Tutor content. This is exemplified by the nature of the tasks that teachers design, the examples used, and the questioning approaches utilized (Biggs, 1999; Zaslavsky, 1995; Redfield & Rousseau, 1981).

Further, we propose that the model presented can be generalized to (math-focused) ITLEs (Slavin, Lake & Groff, 2009). This is because: (1) These learning environments simulate teacher behavior, e.g., through the use of provision of real time feedback, formative hints, individualized pacing of student progress, etc. (2) They typically focus on a bounded math concept (e.g., proportional reasoning, etc), that are operationalized to have clear starting and ending points, so that verification of the acquisition of the requisite skill is possible. (3) These systems typically focus on *practice*, i.e., "encoding, strengthening and proceduralizing knowledge" (Ritter et al., 2007, p. 250; Scholer et al, 2000).

In the next section, we present the two main model constructs – teachers' roles and learning outcomes, and then describe how teachers' roles are predictive of the learning outcomes. This will then be followed by a description of how the supplementary, third set of model constructs influence the degree and quality of the predicted learning outcomes.

3.1 Mathematical Proficiencies

Using the Five (interrelated) Mathematical Strands of Proficiency as the framework, we identify three possible target proficiencies or learning outcomes for the CT-based robotics curriculum foregrounding proportional reasoning at middle school level, i.e., RAP implementations. These are:

(I) Procedural Fluency;
(II) Strategic Competence;
(III) Conceptual Understanding

The importance of these mathematical proficiencies is that a focus on any of these proficiencies determines the learning approaches of particular classrooms or learning sites. This means that the questioning approaches utilized and subsequent interaction patterns with students, the examples used, and the nature of the tasks that teachers design would be aligned to a certain degree with the identified target proficiencies (Kane, Sandretto, & Heath, 2002; Stein & Kim, 2009). In the section below, we describe the three target mathematical proficiencies.

3.1.1 Procedural Fluency

This strand refers to "skill in carrying out [mathematical algorithms or] procedures flexibly, accurately, efficiently, and appropriately" (NRC, 2001, p. 116). This is analogous to the procedures without connections description of a learning approach that is focused on improving performance through practice that is devoid of relating the procedures to the underlying concepts (Stein & Kim, 2009), i.e., reproductive thinking (NRC, 2001, p. 126). In the RAP, procedural fluency is achieved when students complete the Tutor-based Curriculum and are able to use both unit rate and scalar factor strategies to efficiently and flexibly solve problems focusing on robotics motion. This is measured by gains in pre/post assessment on student's ability to reason proportionally by focusing on the number of accurate solutions/strategies employed by students.

Why is procedural fluency a desirable learning outcome? A focus on this approach can lead to improvement in performance on related algorithmic or procedure-focused assessments e.g., some standardized tests, hence the prevalence of teaching to the test: "a carefully developed procedure can be a powerful tool for completing routine tasks". Moreover, some algorithms or skills are representative of functional concepts (NRC, 2001, p. 121) More specifically, the use of multiple strategies, e.g. unit rate and scalar factor, is often an indication or greater progression in student ability to reason proportionally, compared to students who are not able to solve proportionality-focused problems or only use the cross-multiplication method (Boston, Smith & Hillen, 2003). Ideally, subsequent instruction should build on the acquired fluency, as a "certain level of skill is required to learn many mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding" (NRC, 2001, p. 122).

3.1.2 Strategic Competence

This strand refers to the "ability to formulate, represent, and solve mathematical problems". Essentially, this strand depicts a student's ability to construct a "mental model of the variables and relations described in a problem. For example in the RAP, a student displaying strategic competence would not only be able to solve proportional reasoning based problems, using unit rate and scalar factor strategies, but would also, for instance, be able relate the size/circumference of a robot(s) wheels to relative distances moved (e.g., Silk & Schunn, 2011; Schwartz & Moore, 1998). Strategic competence within the RAP context is therefore focused on achieving streamlined understanding of a particular topic within a limited context, i.e., mentally modeling or representing how the physical characteristics of a robot(s) influence the mathematical operations performed on it.

The strategic competence proficiency enables productive (NRC, 2001, p. 126) or mechanistic thinking (Kaplan & Black, 2003; Silk & Schunn, 2011), and is analogous to the procedures with connections approach described in mathematics education literature (Stein, Smith, Henningsen, Silver, 2009). It is also beneficial in helping students derive solutions for non-routine problems. However, achieving strategic competence places certain cognitive and knowledge demands on teachers to enact and/or facilitate instruction at the appropriate degree and quality levels required. This would be explained in Section 5, with respect to the supplementary, third set of model constructs.

3.1.3 Conceptual Understanding

This refers to the "integrated and functional grasp of mathematical ideas", which empowers students "to learn new ideas by connecting those ideas to what they already know." (NRC, 2001, p. 118). In the RAP, achieving conceptual understanding, i.e., becoming a proficient proportional reasoner (Lamon, 2007, Ben-Chaim et al, 1998; Lesh, Post & Behr, 1988), means that such students would be able to solve a variety of proportional problems (missing value, numerical comparison, qualitative), using a variety of strategies in a flexible manner, i.e. using the most efficient strategy per situation, in both robotics and non-robotics contexts.

In contrast to the strategic competence proficiency, which emphasizes streamlined connections, places limited demands on teachers and students, and is typically limited to familiar (i.e. robotics) contexts, conceptual understanding requires a much greater degree and quality of understanding. It focuses on comprehensive understanding of the relevant concepts, e.g., for proportionality – ratio, invariance, covariance, requiring interleaved connections throughout the curriculum. This is analogous to "doing mathematics," and hence often places rigorous demands on teacher mathematical knowledge and pedagogical efficiency (Stein & Smith, 2010) – see Section 5.

In summary, it is important to point out that the three proficiencies are not necessarily linear or mutually exclusive. Instead, they are mutually reinforcing. They have been presented here as more or less independent learning outcomes because of the peculiar affordance of the Tutor ITSLE, which prioritizes procedural fluency and by default, is teacherless. In the next section, we present our conceptualization of the teacher's role, and how the various expressions of the teacher's role interact with the three target mathematical proficiencies to predict teacher models and associated learning outcomes during RAP implementations.

3.2 Teacher Role

The research evidence indicates there is consensus that teacher's perceptions of their roles significantly influence their teaching practice (Kane, Sandretto, & Heath, 2002; Pajares, 1992). For example, the Tutor designers (Anderson et al, 1995) indicated that "students are not mature enough to simply show up at a teacherless class and learn " (p. 200), and that effective Tutor-based implementations would require flexibility, so as to account for teachers' "needs and beliefs about instruction" (p. 192).

In view of these, we conceptualize teacher's role within the context of the RAP as having two dimensions: **(1)** Who does (most) of the teaching, and **(2)** the goal of instruction. So one dimension is when the human teacher perceives the Tutor as being largely responsible for the teaching, i.e., the Tutor is the predominant teacher. Hence the teacher assumes a minimalist or reactive role, allowing the Tutor to drive the bulk of the teaching. The Tutor is assumed to have the requisite instructional capability or fidelity. This is depicted as *TR1a*. In contrast, the teacher may view his/her role as that of a co-teacher with the Tutor. Hence the teacher is proactive in directing the learning process, including within the Tutor, and the designing or facilitation of appropriate instructional activities outside the Tutor to further augment student learning. This is depicted as *TR1b*.

The second dimension of how teachers perceive their roles relate to what they perceive to be the goal of instruction during RAP implementations. Just as student goals are predictive of learning focus, e.g., a focus on performance or understanding (Pintrich, Conley & Kempler, 2003) and are domain specific, we propose that teacher goals would be similarly predictive of learning outcome facilitation focus within specific subject domains. There are two broad possibilities. The teacher may perceive the goal of instruction as helping students perform the requisite procedures flexibly and efficiently. Therefore, the teacher goal here is to support reproductive thinking, and hence the target proficiency is *procedural fluency*. This is depicted as *TR2a*. In contrast, other teachers may perceive their roles as helping students develop interconnections between the various procedures and constituent concepts or variables. As the goal of instruction is the facilitation of the understanding of the relevant concepts, the teacher goal is the support of adaptive or conceptual thinking. Hence the target proficiency is either *strategic competence* (depicted as *TR2b*) or *conceptual understanding* (depicted as *TR2c*), so the goal of instruction is the facilitation of the understanding of the relevant concepts.

4. PREDICTING TEACHER MODELS AND ASSOCIATED LEARNING OUTCOMES

We have presented the target proficiencies that are accessible during RAP implementations as procedural fluency, strategic competence and conceptual understanding. We have also described the two dimensions or expressions of the teacher's role - teachers' views of their instructional role, compared to that of the Tutor (i.e., *TR1a* and *TR1b*), and the goal of instruction (i.e., *TR2a*, *TR2b*, and *TR2c*). In this section we propose that that four possible teacher-dependent learning outcome models may be predicted during RAP implementations. This is based on the interaction of the various dimensions of the teacher's role with the three target mathematical proficiencies. The four predicted teacher-based learning outcome models are: Reproducer, Reproducer Plus, Adapter, and Adapter Plus (Table 1).

Table 1. Description of the teacher models, i.e., teacher-dependent learning outcome models.

<table>
<tr><th colspan="2">Teacher Role</th><th>Predicted Learning Outcomes</th><th>Influencing Factors (Qualitative)</th></tr>
<tr><td>Who teaches?</td><td>Focus of instruction?</td><td>Predicted outcomes with respect to the target proficiencies (NRC, 2001): Procedural Fluency, Strategic Competence, & Conceptual Understanding</td><td rowspan="5">These influence the degree and quality (i.e., Fidelity – Low, Medium, High) of the predicted learning outcomes:
○ What is the teacher's (TK) knowledge? [TK1] [TK2] [TK3] [PD]
• What is the level of student maturity and familiarity with computers?
• To what degree does the learning site support teaching and learning?</td></tr>
<tr><td>TR1a
(Tutor)</td><td>TR2a
Procedural Fluency</td><td>Model 1: Reproducer</td></tr>
<tr><td rowspan="3">TR1b
(Tutor + Teacher)</td><td>TR2a
Procedural Fluency</td><td>Model 2: Reproducer Plus</td></tr>
<tr><td>TR2b
Strategic Competence</td><td>Model 3: Adapter</td></tr>
<tr><td>TR2c
Conceptual Understanding</td><td>Model 4: Adapter Plus</td></tr>
</table>

4.1 Reproducer

Model 1: Reproducer (TR1a, TR2a): This describes a model where the human teacher views the Tutor as being the teacher, and the goal of instruction as reproduction, i.e., helping students perform or reproduce procedures efficiently. So when students struggle during any phase of the Tutor-based curriculum, the approach is to direct the students to the appropriate instructions within the Tutor, so as to redirect and re-focus their learning progression. This approach is more or less reactive, as the teacher does not actively promote engagement with learning outside what the Tutor offers. As the Tutor, whose content is procedure-focused, largely drives the instruction and subsequent learning, *the student learning outcome facilitated is procedural fluency*.

4.2 Reproducer Plus

Model 2: Reproducer Plus (TR1b, TR2a): This describes a model where teachers are proactive in their approach to teaching the Tutor—based content. They recognize that students would often need instructional inputs, external to the Tutor content, in order to overcome learning challenges; and are willing to provide these inputs as needed, using appropriate instructional strategies. However, the focus is on reproductive thinking, helping students perform or reproduce procedures efficiently, e.g., the adoption of "reproductive questioning patterns" (Tienken, Goldberg, & DiRocco, 2009). Hence *the learning outcome facilitated is procedural fluency*.

4.3 Adapter

Model 3: Adapter (TR1b, TR2b): This describes a model where teachers, as in the Reproducer Plus model, are also proactive in their approach to teaching the Tutor—based content. But unlike those within the Reproducer Plus model, they focus on helping students develop adaptive or conceptual thinking in a streamlined strategic domain that is relevant to the Tutor or instructional content. Hence *the learning outcome facilitated is* *strategic competence*, and student understanding is construed to be limited to a particular domain or familiar context.

4.4 Adapter Plus

***Model 4*: Adapter Plus (TR1b, TR2c):** The main difference between the Adapter and Adapter Plus models is that teachers within the latter model view their role as helping students develop comprehensive understanding of the related concept(s) in both familiar and non-familiar contexts. Hence *the learning outcome facilitated is conceptual understanding*. This role operationally places more demands on teacher knowledge and capabilities.

In summary, the four models represent different teacher models and the associated teacher-facilitated learning outcomes. By focusing on the teacher model conceptualizations described, we would be able to align the subsequent design and development of the educative guides with the respective target teacher models.

5. FACTORS INFLUENCING THE DEGREE AND QUALITY OF PREDICTED LEARNING OUTCOMES

In this section, we present a set of supplementary constructs that may influence the degree and quality (i.e., fidelity) of the predicted learning outcomes earlier described. For example, a reproducer learning outcome model may be at a low, medium or high fidelity, based on the interplay of these constructs, which include *teacher knowledge*, as well as *student* and *learning site* attributes.

5.1 Teacher Knowledge

> *It is self-evident that teachers cannot teach what they do not know*
> (National Mathematics Advisory Panel, 2008, xxi)

Teacher knowledge is construed as consisting of the minimum skills and abilities required for instructors to enact the Tutor-based RAP curriculum, in order to facilitate the three learning outcomes of procedural fluency, strategic competence, and conceptual understanding. Although there are several kinds of teacher knowledge, e.g., practical knowledge and content knowledge (Schneider & Plasman, 2011; Shulman, 2006), we specifically focus on Pedagogical Content Knowledge (PCK), which is "a knowledge of teaching that is domain specific; it is what teachers know about their subject matter, and how to make it accessible to students" (Schneider & Plasman, 2011, p. 534; see also Carter, 1990; Gess-Newsome & Lederman, 1999; Shulman, 1986). This is due to the domain specificity of proportional reasoning that, by extension, would require appropriate PCK levels from the teachers involved. This is especially relevant if PCK is viewed as "an amalgamation or transformation (not an integration) of subject matter, pedagogical and context knowledge" (Schneider & Plasman, 2011, p. 533).

Using this PCK-dependent schema to delineate teacher knowledge (TK), three categories of teachers, based on expected PCK skill levels in proportional reasoning, may be proposed:

TK1 ('The Numerical Graduate'): This represents the lowest level of TK (PCK) required to teach proportional reasoning in a way that would facilitate understanding-based outcomes, i.e., strategic competence or conceptual understanding. 'Numerical graduate' is used here to refer to a teacher or informal instructor with a college degree in a numerical discipline (e.g., Physics, Engineering, Computer Science, etc). However instructors at the TK1 level do not have the *generalized, disciplinary repertoire* of skills and knowledge with respect to teaching mathematical content. This knowledge base may be acquired through having a degree in the content area (i.e, mathematics), further content training and regular instructional experience.

TK2 ('The Math/Science Teacher'): This represents the intermediate PCK skill level. The 'math science/teacher' is used here to describe teachers or informal instructors who have a math or science degree, and/or certification. Further, instructors at this TK level have acquired a generalized repertoire - through experience (i.e., regular instructional opportunities) and focused training - of what it means to teach mathematical content.

TK3 ('The Proportionality Teacher'): This is the target ideal or advanced PCK skill level. The 'proportionality teacher' is used to refer to instructors with a math or science degree, and/or math certification, but with additional preparation or focus on proportional reasoning. This is in addition to having acquired a generalized repertoire of knowledge and skills with respect to teaching mathematics.

We propose that the fidelity of the predicted learning outcomes (Table 1) would be influenced by the PCK skill levels of the participating teachers (see Kagan, 1992, pp. 140, 160). We use the term "teacher" as inclusive of both certified public/private school teachers and instructors, typically (paid) volunteers, who teach at informal learning sites. Further, teacher knowledge would be a major consideration in the design of educative guides. In addition, we anticipate that a critical driver for the enhancement and formative development of teacher knowledge is the richness and effectiveness of the *professional development* (PD) opportunities that the different categories of teachers are exposed to. Hence PD is construed for this purpose as an enrichment source for teacher knowledge (Van Driel & Berry, 2012). The PCK skills' strengthening would enable teachers to adapt to "their local contexts and the needs of their students" (p. 27; see also Kagan, 1992).

Therefore bespoke PD programs (to be supplemented by the educative guides during implementations) would play a significant role in advancing TK skill levels from the introductory or initial TK1 category to the ideal TK3 category. This is to ensure that the human teachers for the Tutor have the necessary knowledge and tools to facilitate instruction that would enable meaning-making and conceptual relations, i.e., to actualize the *adapter* and *adapter plus* teacher-facilitated learning outcome models).

5.2 Student and Learning Site Attributes

We conceptualize *student attribute*s as consisting of two dimensions: (1) Student maturity, and (2) familiarity with computers (e.g., self-efficacy with computers, previous exposure to technology, etc; see Moos & Azevedo, 2009, p. 587). For example, the designers of the Tutor partially attributed the success of the initial, exploratory study on the Tutor effectiveness to the participation of "relatively mature students" who were "generally familiar with computers." (Anderson et al., 1995, p. 199). Hence we propose that student maturity and familiarity with computers would to a certain degree influence the fidelity of learning outcomes at RAP implementation sites.

Learning site attributes are conceptualized as those learning site factors (e.g., Van Driel & Berry, 2012, p. 27; Kagan, 1992, p. 152) that are disruptive (i.e., not supportive) of high quality, enacted as intended RAP implementations. These factors include the ongoing technical infrastructure and support that would be available during the duration of the implementation, and whether there would be standardized testing (e.g., PSSA) during the timeline of the implementation and how this would subsequently affect the integrity or intensity of implementation. The others include whether the classes would be offered as elective or required, which in turn can significantly influence student motivation, attendance or retention rates.

6. CONCLUSION & FUTURE WORK

The model presented, based on anticipated real world learning conditions and constraints (Opfer & Pedder, 2011), explains how teacher's role, together with teacher capacity (including learning opportunities offered in professional development), student and other relevant learning site attributes, need to be aligned to facilitate procedural fluency, strategic competence and conceptual understand learning outcomes.

A next step will involve verifying the model in naturalistic settings. We will investigate the fidelity of the predicted learning outcomes by collecting/analyzing relevant data at both formal and informal learning sites between summer 2011 and Spring 2012. There are currently five different RAP implementation sites (Table 2). The methodology is predicated on a mixed-methods approach, and includes professional development and formal/informal site implementation observations, surveys on teacher's role, paper and pencil test of teacher knowledge, student surveys on proportional reasoning ability, and teacher interviews. Data analysis is currently underway.

Another step involves the development of teacher guides. In this paper, we have elaborated the heuristics for the development of teacher educative materials in several ways. First, we have expanded the universe of learning environments to include ITSLEs, environments that have been called essentially "teacherless." In so doing, we have opened up a whole new line of design work surrounding a rapidly growing segment of education materials. We have also begun to articulate some of the challenges that are unique to designing teacher materials for ITSLEs, namely the need to articulate the role of the teacher with respect to the tutor. In essence, there is another "teacher in the room," and, thus, the teacher has to design a role that not only takes into account students, but also the teaching environment created by the ITSLE.

Table 2. Description of the RAP implementation sites.

	Site Type	Implementation Duration	Grade Level	Class Type	# Teachers	# Students
Site 1	Informal (summer camp)	3 weeks?	4th – 7th	Summer Camp (voluntary enrolment)	2	12
Site 2	Formal (public school)	2 days	6th – 8th	All-day intense schedule over two days	1	50
Site 3	Formal (charter school)	4 weeks	6th – 7th	Elective (students have other options)	1	13
Site 4	Formal (charter school)	6 weeks	7th	Incorporated into regular math class	1 (+ 1 teacher assistant)	19
Site 5	Formal (charter school)	5 weeks	7th – 8th	Incorporated into regular math class	2 (+ 1 teacher assistant)	87

Second, we have introduced the problem faced by teachers who want to teach for higher-level student understandings but are constrained by base student materials that aim to primarily develop procedural fluency. Here, we present the problem as associated with the use of Tutors, but some of the same challenges are faced by teachers who are using textbooks that are primarily focused on procedural understanding. A mismatch between base student materials and teacher goals presents a host of new issues that the designers of educative teacher materials have not yet taken up.

Finally, we propose to design educative teacher materials that correspond to four different teacher models. Aligning the design of the educative guides with respect to these teacher models constitutes an important step forward in research and theory on educative teacher materials which, to date, have been based on an implicit model of one particular kind of teacher's role. Specifically, most of the extant design and research work surrounding educative teacher materials has assumed a default position of the TR2c-based teacher role model, i.e., adopting the goal of a highly teacher-facilitated development of students' conceptual understanding of the topic under investigation. As such, the materials and the heuristics guiding most extant design are primarily aimed at a single type of learning environment. In contrast, our work has the potential to take into consideration questions such as: Educative teacher materials for whom?; Toward what end?; and, given our identification of the supplementary factors, Under what conditions? This degree of differentiation has not previously appeared in the educative materials literature, to our knowledge.

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